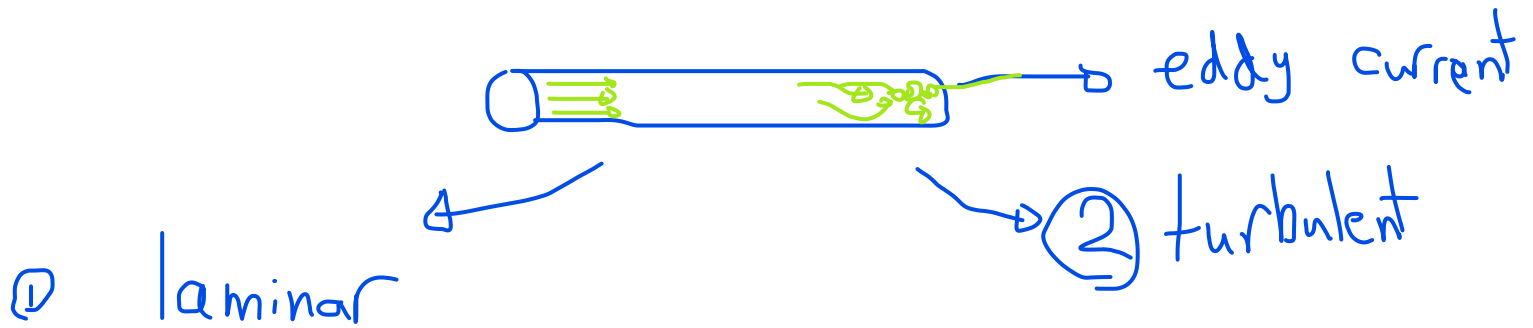


Chapter (10)

10.8

Dynamics : motion of fluids

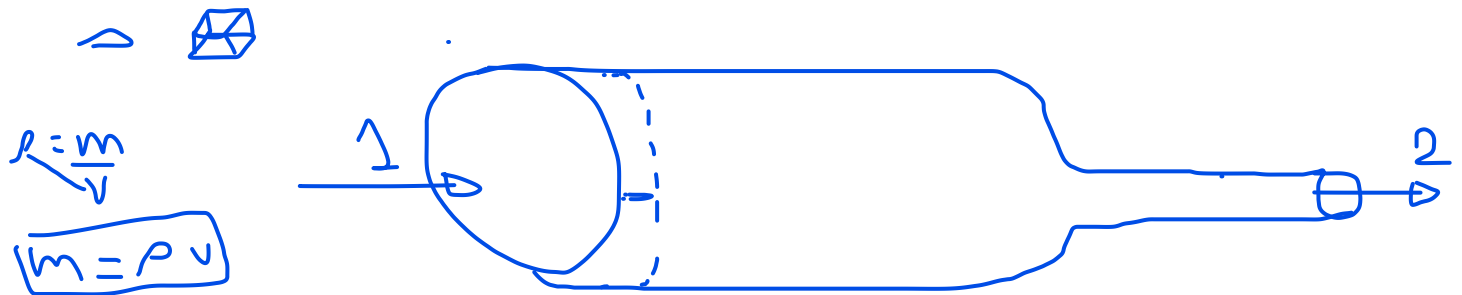
Hemodynamics : , , , water



viscosity : friction between layer

→ Equation of continuity:

- fluid $\neq$ compressible
- laminar



$$\frac{\Delta V_1}{\Delta t} = \frac{\Delta V_2}{\Delta t} \Rightarrow \frac{m_1}{\Delta t} = \frac{m_2}{\Delta t}$$

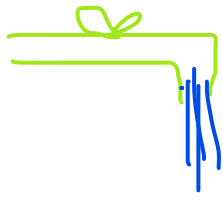
Volume flow rate

because ρ is constant

$$\frac{A_1 \Delta \ell_1}{\Delta t}$$

$$= \frac{A_2 \Delta \ell_2}{\Delta t}$$

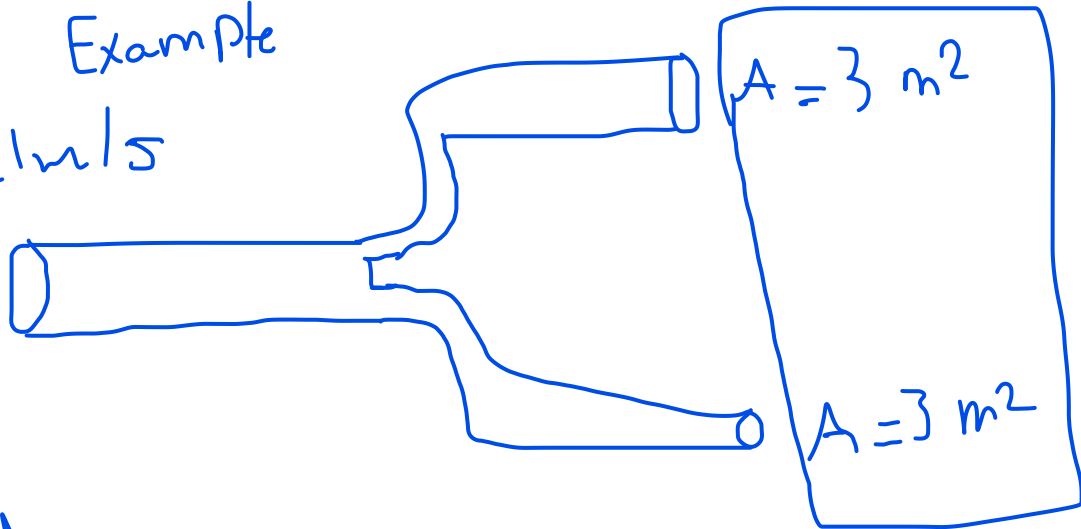
$$\Rightarrow A_1 v_1 = A_2 v_2$$



Example

$$v = 1 \text{ m/s}$$

$$\underline{A_1 = 8 \text{ m}^2}$$



find v_2 $n \times A$

$$A_1 v_1 = A_2 v_2$$

$$8 \times 1 = 2(3) \underline{v_2}$$

10) Water is flowing at 4.0 m/s in a circular pipe. If the diameter of the pipe decreases to 1/2 its former value, what is the velocity of the water downstream?

- a) 1.0 m/s b) 2.0 m/s c) 4.0 m/s d) 8.0 m/s e) 16 m/s

$V = 4 \text{ m/s}$ $\left(\frac{\Delta V_1}{\Delta t} \right)$ $\frac{4 \text{ m/s}}{4 \times}$ $\rightarrow ?$

$A_1 V_1 = A_2 V_2 \rightarrow \frac{\Delta V_2}{\Delta t}$

$\frac{(2 \times \pi r^2) 4}{x^2 \pi r^2} = \frac{(x^2 \pi r^2) (??)}{x^2 \pi r^2}$

$4 \times 4 = 16$
 $\underline{16} = V_2$

$1 \text{ m}^3 = 100 \text{ cm}^3$

Q11) The cross-sectional area of the aorta is 2 cm² and blood flows through it at 40 cm/s. The mass flow rate (in grams/s) of blood through the aorta is: (Assume density of blood to be 1059 kg/m³)

- A) 0.1 B) 100 C) 84.7 D) 8470 E) 1059

$1 \text{ m}^3 = 1000000 \text{ cm}^3$

$\frac{\Delta V}{\Delta t}$

$\frac{m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{A \rho v}{2 \times 40}$

$(10^{-3} \times 1059) \times 80$

$\frac{\text{kg} \times 1000}{\text{m}^3 \times 10^6} \rightarrow \text{cm}^3$

$\frac{\text{kg}}{\text{m}^3} = \frac{\text{g}}{\text{cm}^3} \times \frac{1000}{10^6}$

12. 2 Liters/s of water enter a pipe of radius 1 cm. The speed of the water inside the pipe (in m/s) is:

A) 6.37

B) 3.71

C) 0.28

D) 8.46

E) 12.7

$$1 \text{ m}^3 = 1000 \text{ L}$$

$$Av = \frac{\Delta V}{\Delta t}$$

$$\text{(m)}^2 \frac{\text{m}}{\text{s}} = \frac{\text{m}^3}{\text{s}}$$

$$\frac{(0.01)^2 \pi v}{\pi (0.01)^2} = \frac{0.002 \text{ m}^3/\text{s}}{(0.01)^2 \pi}$$

10.9

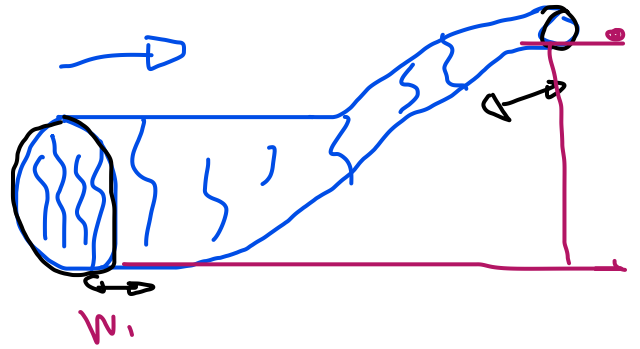
Bernoulli's equation

$$W = F d \cos \theta$$

$$W_{\text{total}} = \Delta K$$

$$W_{\text{weight}} = -\Delta P$$

$$P = \frac{F}{A}$$



$$W_1 = F \Delta l_1 = P A_1 \Delta l_1$$

$$W_2 = -F \Delta l_2 = -P A_2 \Delta l_2$$

$$W_p = -h g m$$

$$P A_1 \Delta l_1 - P A_2 \Delta l_2 - h g m = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$P v_1 - P v_2 - y_2 g m + y_1 g m = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$P_1 - P_2 - y_2 g \rho + y_1 g \rho = \frac{1}{2} v_2^2 \rho - \frac{1}{2} v_1^2 \rho$$

$$P_1 + y_1 g \rho + \frac{1}{2} \rho v_1^2 = P_2 + y_2 g \rho + \frac{1}{2} \rho v_2^2$$

11) Water pressurized to 3.5×10^5 Pa is flowing at 5.0 m/s in a horizontal pipe which contracts to $1/3$ its former area. What are the pressure and velocity of the water after the contraction?

- a) 4.5×10^5 Pa, 1.5 m/s b) 3.0×10^5 Pa, 10 m/s c) 3.0×10^5 Pa, 15 m/s
d) 2.5×10^5 Pa, 15 m/s e) 5.5×10^5 Pa, 1.5 m/s

$$\frac{1}{3} A_1 = A_2$$

$$A_1 v_1 = A_2 v_2$$

$$A_1 (5) = \frac{1}{3} A_1 v_2$$

$$v_2 = 15 \text{ m/s}$$

$$P_1 + \cancel{y_1 g} + \frac{1}{2} \rho v_1^2 = P_2 + \cancel{y_2 g} + \frac{1}{2} \rho v_2^2$$

$$P_2 = 2.5 \times 10^5 \quad 3.5 \times 10^5 + \frac{1}{2} (1000) 5^2 = P_2 + \frac{1}{2} (1000) 15^2$$

11. Air flowing horizontally with a speed v over the flat roof of a building reduces the pressure on the roof by an amount ΔP . What is the pressure reduction if the speed of the air is $3v$? Assume that the air was still initially.

- A) 0 B) $4\Delta P$ C) $9\Delta P$ D) $\Delta P/9$ E) $\Delta P/4$

$$v_1 = 3v_2$$



$$P_1 + \cancel{y_1 g} + \frac{1}{2} \rho v_1^2 = P_2 + \cancel{y_2 g} + \frac{1}{2} \rho v_2^2$$

$$P_1 + \frac{1}{2} \rho (3v_2)^2 = P_2 + \frac{1}{2} \rho v_2^2$$

Applications

Torricelli's theorem

Find the speed v_1 of the water out of the small shown opening.

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

Note: $A_2 \gg A_1 \Rightarrow v_2 \sim 0$

$$(A_1 v_1 = A_2 v_2) \cdot$$

$$P_1 = P_2 = P_{\text{atm}}$$

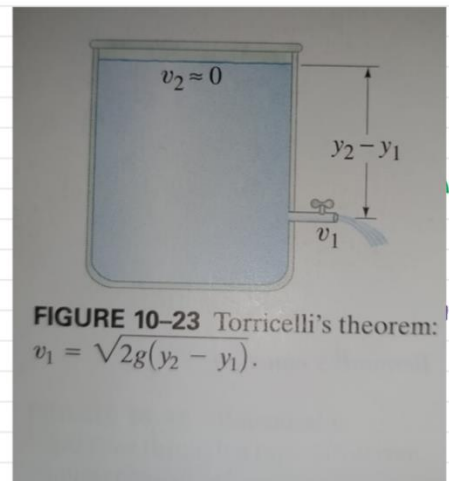


FIGURE 10-23 Torricelli's theorem:
 $v_1 = \sqrt{2g(y_2 - y_1)}$.

$$\therefore \cancel{P_{atm}} + \rho g y_1 + \frac{1}{2} \rho v_1^2 = \cancel{P_{atm}} + \rho g y_2 + \frac{1}{2} \rho v_2^2 = 0$$

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 \approx 0$$

$$\therefore \frac{1}{2} \rho v_1^2 = \rho g (y_2 - y_1) = \rho g h$$

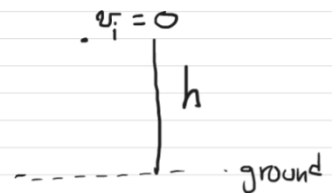
$$\therefore v_1^2 = 2gh \Rightarrow v_1 = \sqrt{2gh}, \quad h = y_2 - y_1$$

Compare: A stone is released from rest from a height h above the surface of the ground. Find its speed just before it hits the ground.

$$\downarrow a = g \quad v_f^2 - v_i^2 = 2g(y_f - y_i)$$

$$v_f^2 - 0 = 2gh$$

$$\therefore v_f = \sqrt{2gh}$$

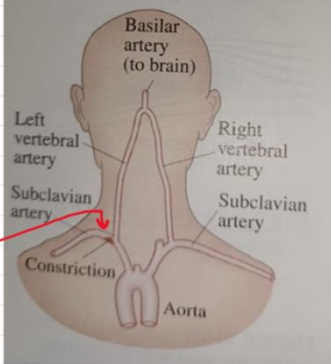


So, the water in the tank acts as if it is in free fall.

Q 62%

Transient Ischemic Attack (TIA)

FIGURE 10-28 Rear of the head and shoulders showing arteries leading to the brain and to the arms. High blood velocity past the constriction in the left subclavian artery causes low pressure in the left vertebral artery, in which a reverse (downward) blood flow can then occur, resulting in a TIA, a loss of blood to the brain.



Small $A \rightarrow$
Large v
 $\rightarrow$ Small
Pressure

This means temporary lack of blood supply to the brain. It causes symptoms such as double vision, headache, weakness of limbs.

blood pressure in right subclavian artery is higher than left subclavian artery $\Rightarrow$ instead of blood going to the brain it is diverted to the left subclavian artery as it has lower pressure.

Lift force on airplane wing : Lift force causes plane to fly.

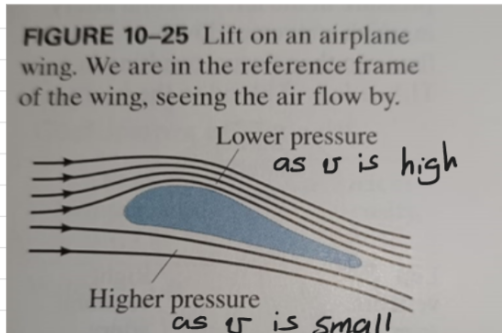
It is due to the difference in pressure.

below wing : low $v \rightarrow$ high P .

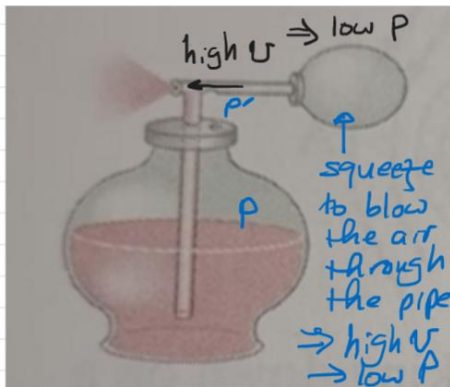
above wing : high $v \rightarrow$ low P .

$\Rightarrow$ resultant lift force acts on the wing upwards.

Remember: $F = P A$.



Perfume Atomizer



Air is blown at high speed in the horizontal tube $\Rightarrow$ high $v \Rightarrow P'$ is low.

P' is lower than the pressure inside the perfume container $\Rightarrow$

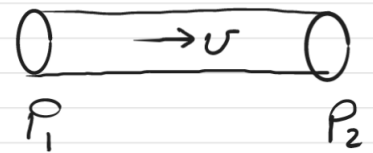
perfume is pushed upwards through the vertical tube and then outside.

10-12] Flow in tubes: Poiseuille's Equation, Blood Flow

A non-viscous fluid can flow in a horizontal tube without applying a force.

When the fluid is viscous, the viscosity plays the role of friction and impedes the flow of the fluid layers.

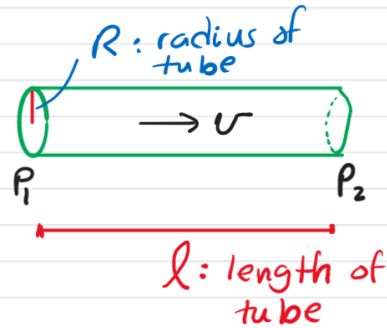
For a viscous fluid to flow in the tube a pressure difference between the two ends of a level tube is necessary for the steady flow of any real (viscous) fluid.



Poiseuille studied the factors that affect the flow rate of an incompressible fluid undergoing laminar flow in a cylindrical tube. His equation known as Poiseuille's equation is given by:

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8 \eta l}$$

$P_1 - P_2$: pressure difference between the two ends of the tube.



R : Inside radius of the tube

l : length of the tube.

η : coefficient of viscosity, which has units of $\frac{\text{N} \cdot \text{s}}{\text{m}^2} \equiv \text{Pa} \cdot \text{s}$

Q : volume flow rate which has units of volume/unit time which is m^3/s in the SI system

Note that $Q \propto R^4 \Rightarrow$ when $R \rightarrow \frac{R}{2}$

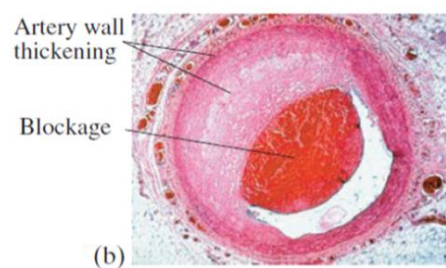
$$\therefore Q \rightarrow Q/16$$

Blood flow in the arteries is not 100% streamline flow, it can also be turbulent flow $\Rightarrow$ Poiseuille's equation applies approximately to blood flow in the arteries, veins, capillaries.

Arteriosclerosis (thickening of the artery walls) cause a reduction in the radius R .

So, to maintain the same flow rate Q unchanged an increase in blood pressure gradient

$\frac{P_1 - P_2}{l}$ is required.



Therefore, the heart must work harder to maintain the same flow rate. Therefore, an increase in blood pressure is an indication that the heart is working harder and the blood flow rate is reduced.