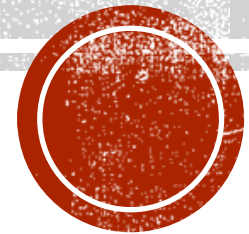


CHAPTER 30

Done by: Abdelhadi Okasha



- 30-1 Structure and Properties of Nucleus
- 30-3 Radioactivity
- 30-8 Half-Life and Rate of Decay
- 30-9 Calculations Involving Decay Rates and Half-Life



30—1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

- The atom is made of electrons moving around the nucleus
- nucleus is made up of two types of particles: protons and neutrons

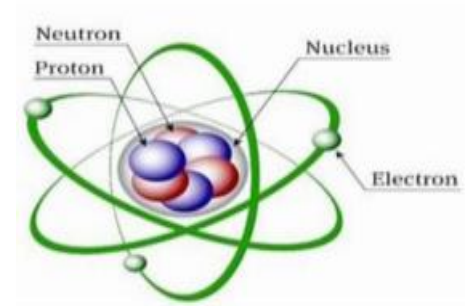
The proton has a positive charge ($= +e = +1.60 \times 10^{-19} \text{ C}$, the same magnitude as for the electron) and its mass is measured to be

$$m_p = 1.67262 \times 10^{-27} \text{ kg.}$$

The **neutron**, whose existence was ascertained in 1932 by the English physicist James Chadwick (1891–1974), is electrically neutral ($q = 0$), as its name implies. Its mass is very slightly larger than that of the proton:

$$m_n = 1.67493 \times 10^{-27} \text{ kg.}$$

These two constituents of a nucleus, neutrons and protons, are referred to collectively as **nucleons**.



	Mass	Charge
Electron	9.110×10^{-31}	-1.6×10^{-19}



30—1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

- Although a normal hydrogen nucleus consists of a single proton alone, the nuclei of all other elements consist of both neutrons and protons. The different nuclei are often referred to as nuclides

We refer to a given nuclide (nucleus) as:

Atomic number (Z):

number of protons in the nucleus.

mass number $\rightarrow A$

atomic number $\rightarrow Z$

\times

N

neutron number

Neutron number (N)

number of neutrons in the nucleus.

Mass number: the sum of proton and neutron numbers in the nucleus

$$A = Z + N$$



30—1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

For a particular type of atom (say, carbon), nuclei are found to contain different numbers of neutrons, although they all have the same number of protons. For example, carbon nuclei always have 6 protons, but they may have 5, 6, 7, 8, 9, or 10 neutrons. Nuclei that contain the same number of protons but different numbers of neutrons are called **isotopes**. Thus, $^{11}_6\text{C}$, $^{12}_6\text{C}$, $^{13}_6\text{C}$, $^{14}_6\text{C}$, $^{15}_6\text{C}$, and $^{16}_6\text{C}$ are all isotopes of carbon. The isotopes of a given element are not all equally common. For example, 98.9% of naturally occurring carbon (on Earth) is the isotope $^{12}_6\text{C}$, and about 1.1% is $^{13}_6\text{C}$. These percentages are referred to as the **natural abundances**.[†] Even hydrogen has isotopes: 99.99% of natural hydrogen is ^1_1H , a simple proton, as the nucleus; there are also ^2_1H , called **deuterium**, and ^3_1H , **tritium**, which besides the proton contain 1 or 2 neutrons. (The bare nucleus in each case is called the **deuteron** and **triton**.)



Isotones: nuclides that have the same number of **neutrons**, but different number of protons. ${}^{12}_{5}\text{B}_7$, ${}^{13}_{6}\text{C}_7$

(same N different Z)

Isobars: nuclides that have the same mass number A . ${}^{40}_{18}\text{Ar}_{22}$; ${}^{40}_{19}\text{K}_{21}$; ${}^{40}_{20}\text{Ca}_{20}$

(same A)



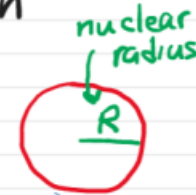
30—1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

Nuclei (nuclides) have roughly spherical shapes. The radius of a nucleus is given

by :

$$R = 1.2 \times 10^{-15} A^{1/3} \text{ meters}$$

↑ mass number
 $A = Z + N$



$$1 \text{ fermi} = 10^{-15} \text{ m}$$

$$R = 1.2 A^{1/3} \text{ fermi}$$

Example: Determine the radius of ^{16}O .

$$R = 1.2 \times 10^{-15} (16)^{1/3} \approx 3.02 \times 10^{-15} \text{ m}$$

$$R \sim 3.02 \text{ fm} \text{ (fm} \equiv \text{fermi)}.$$



EXAMPLE 30-1 **ESTIMATE** **Nuclear sizes.** Estimate the diameter of the smallest and largest naturally occurring nuclei: (a) ${}^1_1\text{H}$, (b) ${}^{238}_{92}\text{U}$.

APPROACH The radius r of a nucleus is related to its number of nucleons A by Eq. 30-1. The diameter $d = 2r$.

SOLUTION (a) For hydrogen, $A = 1$, Eq. 30-1 gives

$$d = \text{diameter} = 2r \approx 2(1.2 \times 10^{-15} \text{ m})(A^{\frac{1}{3}}) = 2.4 \times 10^{-15} \text{ m}$$

since $A^{\frac{1}{3}} = 1^{\frac{1}{3}} = 1$.

(b) For uranium $d \approx (2.4 \times 10^{-15} \text{ m})(238)^{\frac{1}{3}} = 15 \times 10^{-15} \text{ m}$.

The range of nuclear diameters is only from 2.4 fm to 15 fm.

NOTE Because nuclear radii vary as $A^{\frac{1}{3}}$, the largest nuclei (such as uranium with $A = 238$) have a radius only about $\sqrt[3]{238} \approx 6$ times that of the smallest, hydrogen ($A = 1$).

EXAMPLE 30-2 **ESTIMATE** **Nuclear and atomic densities.** Compare the density of nuclear matter to the density of normal solids.

APPROACH The density of normal liquids and solids is on the order of 10^3 to 10^4 kg/m^3 (see Table 10-1), and because the atoms are close packed, atoms have about this density too. We therefore compare the density (mass per volume) of a nucleus to that of its atom as a whole.

SOLUTION The mass of a proton is greater than the mass of an electron by a factor

$$\frac{1.67 \times 10^{-27} \text{ kg}}{9.1 \times 10^{-31} \text{ kg}} \approx 2000.$$

Thus, over 99.9% of the mass of an atom is in the nucleus, and for our estimate we can say the mass of the atom equals the mass of the nucleus, $m_{\text{nuc}}/m_{\text{atom}} = 1$. Atoms have a radius of about 10^{-10} m (Chapter 27) and nuclei on the order of 10^{-15} m (Eq. 30-1). Thus the ratio of nuclear density to atomic density is about

$$\frac{\rho_{\text{nuc}}}{\rho_{\text{atom}}} = \frac{(m_{\text{nuc}}/V_{\text{nuc}})}{(m_{\text{atom}}/V_{\text{atom}})} = \left(\frac{m_{\text{nuc}}}{m_{\text{atom}}} \right) \frac{\frac{4}{3}\pi r_{\text{atom}}^3}{\frac{4}{3}\pi r_{\text{nuc}}^3} \approx (1) \frac{(10^{-10})^3}{(10^{-15})^3} = 10^{15}.$$

The nucleus is 10^{15} times more dense than ordinary matter.



30—1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

Nuclear masses can be specified in **unified atomic mass units (u)**. On this scale, a neutral $^{12}_6\text{C}$ atom is given the exact value 12.000000 u. A neutron then has a measured mass of 1.008665 u, a proton 1.007276 u, and a neutral hydrogen atom ^1_1H (proton plus electron) 1.007825 u.



30—1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

Masses may be specified using the electron-volt energy unit, $1 \text{ eV} = 1.6022 \times 10^{-19} \text{ J}$ (Section 17–4). This can be done because mass and energy are related, and the precise relationship is given by Einstein's equation $E = mc^2$ (Chapter 26). Since the mass of a proton is $1.67262 \times 10^{-27} \text{ kg}$, or 1.007276 u , then 1 u is equal to

$$1.0000 \text{ u} = (1.0000 \text{ u}) \left(\frac{1.67262 \times 10^{-27} \text{ kg}}{1.007276 \text{ u}} \right) = 1.66054 \times 10^{-27} \text{ kg};$$

this is equivalent to an energy (see Table inside front cover) in MeV ($= 10^6 \text{ eV}$) of

$$E = mc^2 = \frac{(1.66054 \times 10^{-27} \text{ kg})(2.9979 \times 10^8 \text{ m/s})^2}{(1.6022 \times 10^{-19} \text{ J/eV})} = 931.5 \text{ MeV}.$$

Thus,

$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}/c^2.$$

The rest masses of some of the basic particles are given in Table 30–1. As a rule of thumb, to remember, the masses of neutron and proton are about $1 \text{ GeV}/c^2$ ($= 1000 \text{ MeV}/c^2$) which is about 2000 times the mass of an electron ($\approx \frac{1}{2} \text{ MeV}/c^2$).

TABLE 30–1
Rest Masses in Kilograms, Unified Atomic Mass Units, and MeV/c^2

Object	Mass		
	kg	u	MeV/c^2
Electron	9.1094×10^{-31}	0.00054858	0.51100
Proton	1.67262×10^{-27}	1.007276	938.27
^1_1H atom	1.67353×10^{-27}	1.007825	938.78
Neutron	1.67493×10^{-27}	1.008665	939.57



30—3 RADIOACTIVITY

- Some nuclei (specially heavy ones) are unstable and emit neutrons, alpha particles, gamma radiation, x-rays ... Etc, to become more stable

The isotopes of uranium $^{235}_{92}\text{U}$ and $^{238}_{92}\text{U}$ are radioactive. Both are naturally occurring nuclides
→ natural radioactivity.

Manufactured radioactive nuclides lead to artificial radioactivity. (Californium)
↑ artificially radioactive manufactured element

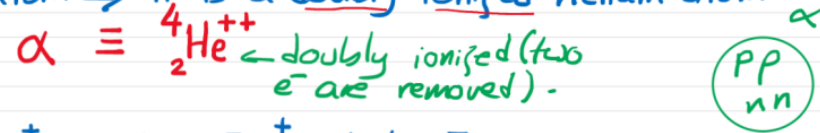


30—3 RADIOACTIVITY

Unstable nuclei emit three main types of nuclear radiation, classified according to their penetrating power:

i) Alpha (α) particles [also can say α -radiation]

An α -particle is made up of 2 protons and 2 neutrons. So it is the nucleus of helium atom \Rightarrow it is a doubly ionized helium atom



ii) β^\pm particles [β^\pm radiation]

β^- is an electron e^- $-1.6 \times 10^{-19} \text{ J}$

β^+ is a positron e^+ $+1.6 \times 10^{-19} \text{ J}$

e^+ is the antiparticle of the e^- .

$m_{e^+} = m_{e^-}$ but $e^+ = +1.6022 \times 10^{-19} \text{ Coulomb}.$

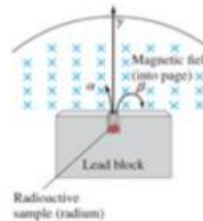


30—3 RADIOACTIVITY

iii) γ -radiation

Very high frequency electromagnetic radiation.

FIGURE 30-4 Alpha and beta rays are bent in opposite directions by a magnetic field, whereas gamma rays are not bent at all.



magnetic field only deflects charged particles. It does not act with a force on neutral particles.



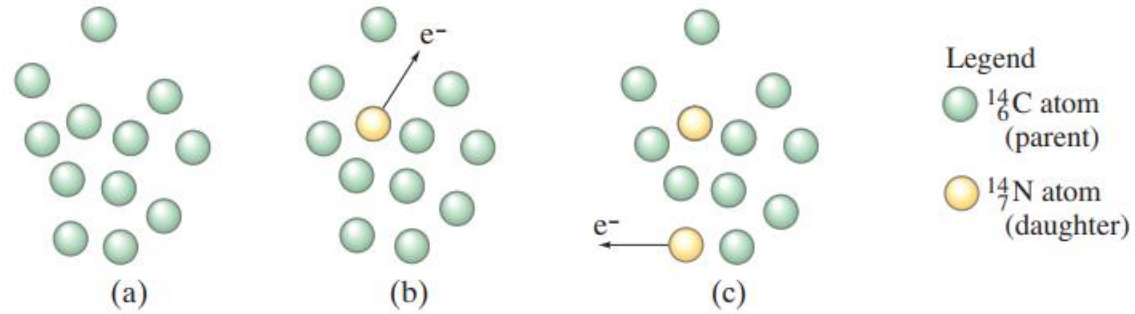
30—3 RADIOACTIVITY

Radiation	Penetrating Power
α	can be stopped using a piece of paper.
β^{\pm}	can be stopped using few mm of aluminum.
γ	has high penetrating power. can pass through several cm of lead.

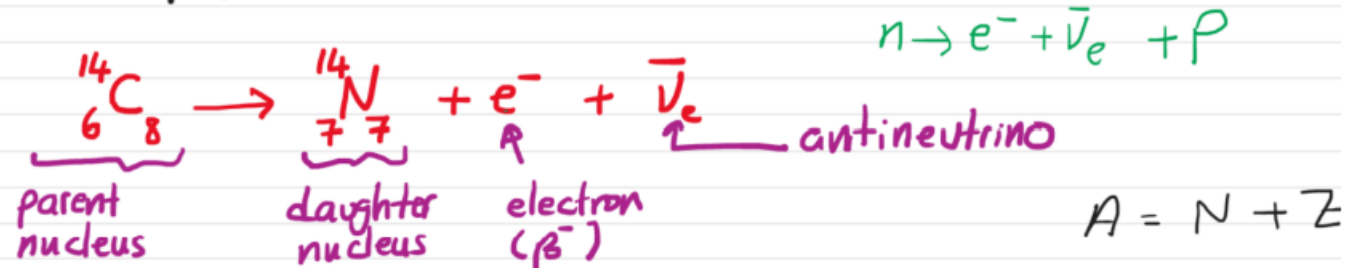


30—8 HALF-LIFE AND RATE OF DECAY

FIGURE 30–9 Radioactive nuclei decay one by one. Hence, the number of parent nuclei in a sample is continually decreasing. When a $^{14}_6\text{C}$ nucleus emits an electron (b), the nucleus becomes a $^{14}_7\text{N}$ nucleus. Another decays in (c).



$^{14}_6\text{C}$ is an unstable nucleus \Rightarrow it decays by emitting β^- particles (e^-) and turns into $^{14}_7\text{N}$ which is stable.



30—8 HALF-LIFE AND RATE OF DECAY

$$N = N_0 e^{-\lambda t},$$

where N_0 is the number of parent nuclei present at any chosen time $t = 0$, and N is the number remaining after a time t . The symbol e is the natural exponential (encountered earlier in Sections 19–6 and 21–12) whose value is $e = 2.718\cdots$. Thus the number of parent nuclei in a sample decreases exponentially in time. This is shown in Fig. 30–10a for the decay of $^{14}_6\text{C}$. Equation 30–4 is called the **radioactive decay law**.

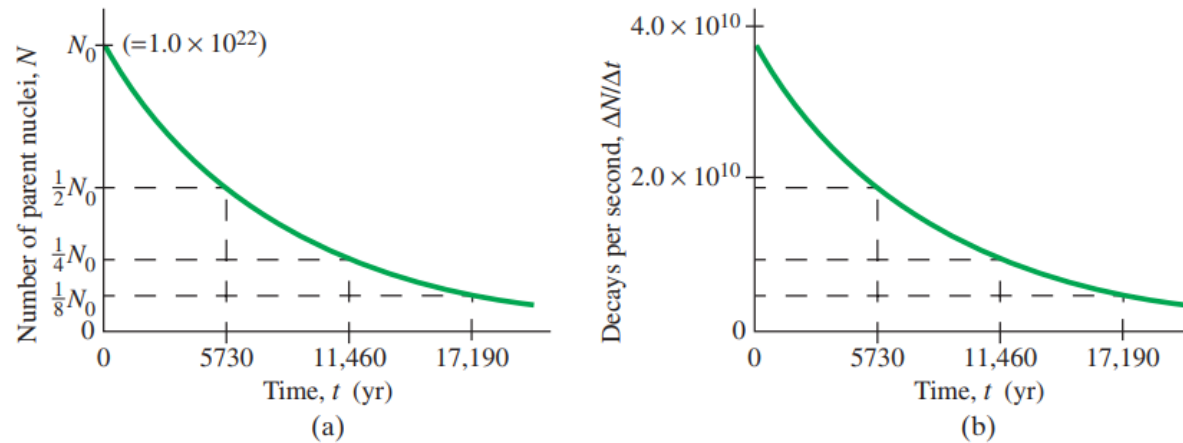


FIGURE 30–10 (a) The number N of parent nuclei in a given sample of $^{14}_6\text{C}$ decreases exponentially. We assume a sample that has $N_0 = 1.00 \times 10^{22}$ nuclei. (b) The number of decays per second also decreases exponentially. The half-life of $^{14}_6\text{C}$ is 5730 yr, which means that the number of parent nuclei, N , and the rate of decay, $\Delta N/\Delta t$, decrease by half every 5730 yr.



30—8 HALF-LIFE AND RATE OF DECAY

Half-Life

The rate of decay of any isotope is often specified by giving its “half-life” rather than the decay constant λ . The **half-life** of an isotope is defined as the time it takes for half the original amount of parent isotope in a given sample to decay.

* Deriving the Half-Life Formula

We can derive Eq. 30–6 starting from Eq. 30–4 by setting $N = N_0/2$ at $t = T_{\frac{1}{2}}$:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{\frac{1}{2}}}$$

so

$$\frac{1}{2} = e^{-\lambda T_{\frac{1}{2}}}$$

and

$$e^{\lambda T_{\frac{1}{2}}} = 2.$$

We take natural logs of both sides (“ln” and “e” are inverse operations, meaning $\ln(e^x) = x$) and find

$$\ln(e^{\lambda T_{\frac{1}{2}}}) = \ln 2,$$

so

$$\lambda T_{\frac{1}{2}} = \ln 2 = 0.693$$

and

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda},$$

which is Eq. 30–6.



30—8 HALF-LIFE AND RATE OF DECAY

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Each radioactive nuclide has its own λ .

large $\lambda \Rightarrow$ small $t_{1/2} \Rightarrow$ fast decay.

small $\lambda \Rightarrow$ large $t_{1/2} \Rightarrow$ slow decay.

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

For ^{238}U : $t_{1/2} \approx 4.5 \times 10^9 \text{ yr} \Rightarrow \lambda = \frac{\ln 2 \times 10^{-9}}{4.5} \text{ yr}^{-1}$

very small $\lambda \Rightarrow$ very large $t_{1/2} \Rightarrow$ slow decay.

For Iodine-131 (^{131}I) (used to treat thyroid gland cancer and also treatment of an overactive thyroid gland).

$$t_{1/2} \approx 8 \text{ days}$$

$$t_{1/2} \approx 8 \text{ days} \Rightarrow \lambda = \frac{\ln 2}{8} \approx 0.087 \text{ day}^{-1}$$

large λ (compared to ^{238}U) \Rightarrow small $t_{1/2} \Rightarrow$ fast decay.



30-8 HALF-LIFE AND RATE OF DECAY

The number of decays per second, or decay rate R , is the magnitude of $\Delta N/\Delta t$, and is also called the **activity** of the sample. The magnitude (always positive) of a quantity is often indicated using vertical lines. The magnitude of $\Delta N/\Delta t$ is written $|\Delta N/\Delta t|$ and it is proportional to N (see Eq. 30-3b). So it too decreases exponentially in time at the same rate (Fig. 30-10b). The activity of a pure sample at time t is

$$R = \left| \frac{\Delta N}{\Delta t} \right| = R_0 e^{-\lambda t}, \quad (30-5)$$

where $R_0 = |\Delta N/\Delta t|_0$ is the activity at $t = 0$.

Equation 30-5 is also referred to as the **radioactive decay law** (as is Eq. 30-4).

The SI unit of activity is the Becquerel (Bq)

$$1 \text{ Bq} = 1 \text{ decay/s or simply } s^{-1}$$

Another more common unit of activity is the curie (Ci)

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays/s}$$



Activity (A) : number of decays per second .
[rate of decay]

$$N = N_0 e^{-\lambda t}$$

$$A = -\frac{dN}{dt} = -[N_0(-\lambda) e^{-\lambda t}]$$

$$\therefore A = \lambda N_0 e^{-\lambda t}$$

$$A = \lambda N$$

$$A = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$$

↑
activity at
time t .

initial activity



30–8 HALF-LIFE AND RATE OF DECAY

* Mean Life

Sometimes the **mean life** τ of an isotope is quoted, which is defined as $\tau = 1/\lambda$. Then Eq. 30–4 can be written $N = N_0 e^{-t/\tau}$, just as for RC and LR circuits (Chapters 19 and 21 where τ was called the time constant). The mean life of an isotope is then given by (see also Eq. 30–6)

$$\tau = \frac{1}{\lambda} = \frac{T_{\frac{1}{2}}}{0.693}. \quad \text{[mean life] (30–7)}$$

The mean life and half-life differ by a factor of 0.693, so confusing them can cause serious error (and has). The radioactive decay law, Eq. 30–5, can then be written as $R = R_0 e^{-t/\tau}$.



30-9 CALCULATIONS INVOLVING DECAY RATES AND HALF-LIFE

EXAMPLE 30-9 **Sample activity.** The isotope $^{14}_6\text{C}$ has a half-life of 5730 yr. If a sample contains 1.00×10^{22} carbon-14 nuclei, what is the activity of the sample?

APPROACH We first use the half-life to find the decay constant (Eq. 30-6), and use that to find the activity, Eq. 30-3b. The number of seconds in a year is $(60)(60)(24)(365\frac{1}{4}) = 3.156 \times 10^7$ s.

SOLUTION The decay constant λ from Eq. 30-6 is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{(5730 \text{ yr})(3.156 \times 10^7 \text{ s/yr})} = 3.83 \times 10^{-12} \text{ s}^{-1}.$$

From Eqs. 30-3b and 30-5, the activity or rate of decay is

$$R = \left| \frac{\Delta N}{\Delta t} \right| = \lambda N = (3.83 \times 10^{-12} \text{ s}^{-1})(1.00 \times 10^{22}) = 3.83 \times 10^{10} \text{ decays/s}.$$

Notice that the graph of Fig. 30-10b starts at this value, corresponding to the original value of $N = 1.0 \times 10^{22}$ nuclei in Fig. 30-10a.

NOTE The unit “decays/s” is often written simply as s^{-1} since “decays” is not a unit but refers only to the number. This simple unit of activity is called the becquerel: $1 \text{ Bq} = 1 \text{ decay/s}$, as discussed in Chapter 31.



30-9 CALCULATIONS INVOLVING DECAY RATES AND HALF-LIFE

EXAMPLE 30-11 A sample of radioactive $^{13}_7\text{N}$. A laboratory has $1.49\text{ }\mu\text{g}$ of pure $^{13}_7\text{N}$, which has a half-life of 10.0 min (600 s). (a) How many nuclei are present initially? (b) What is the rate of decay (activity) initially? (c) What is the activity after 1.00 h ? (d) After approximately how long will the activity drop to less than one per second ($= 1\text{ s}^{-1}$)?

APPROACH We use the definition of the mole and Avogadro's number (Sections 13-6 and 13-8) to find (a) the number of nuclei. For (b) we get λ from the given half-life and use Eq. 30-3b for the rate of decay. For (c) and (d) we use Eq. 30-5.

SOLUTION (a) The atomic mass is 13.0 , so 13.0 g will contain 6.02×10^{23} nuclei (Avogadro's number). We have only $1.49 \times 10^{-6}\text{ g}$, so the number of nuclei N_0 that we have initially is given by the ratio

$$\frac{N_0}{6.02 \times 10^{23}} = \frac{1.49 \times 10^{-6}\text{ g}}{13.0\text{ g}}.$$

Solving for N_0 , we find $N_0 = 6.90 \times 10^{16}$ nuclei.

(b) From Eq. 30-6,

$$\lambda = 0.693/T_{1/2} = (0.693)/(600\text{ s}) = 1.155 \times 10^{-3}\text{ s}^{-1}.$$

Then, at $t = 0$ (see Eqs. 30-3b and 30-5)

$$R_0 = \left| \frac{\Delta N}{\Delta t} \right|_0 = \lambda N_0 = (1.155 \times 10^{-3}\text{ s}^{-1})(6.90 \times 10^{16}) = 7.97 \times 10^{13}\text{ decays/s}.$$

(c) After $1.00\text{ h} = 3600\text{ s}$, the magnitude of the activity will be (Eq. 30-5)

$$R = R_0 e^{-\lambda t} = (7.97 \times 10^{13}\text{ s}^{-1})e^{-(1.155 \times 10^{-3}\text{ s}^{-1})(3600\text{ s})} = 1.25 \times 10^{12}\text{ s}^{-1}.$$

(d) We want to determine the time t when $R = 1.00\text{ s}^{-1}$. From Eq. 30-5, we have

$$e^{-\lambda t} = \frac{R}{R_0} = \frac{1.00\text{ s}^{-1}}{7.97 \times 10^{13}\text{ s}^{-1}} = 1.25 \times 10^{-14}.$$

We take the natural log (\ln) of both sides ($\ln e^{-\lambda t} = -\lambda t$) and divide by λ to find

$$t = -\frac{\ln(1.25 \times 10^{-14})}{\lambda} = 2.77 \times 10^4\text{ s} = 7.70\text{ h}.$$

Easy Alternate Solution to (c) $1.00\text{ h} = 60.0\text{ minutes}$ is 6 half-lives, so the activity will decrease to $(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = (\frac{1}{2})^6 = \frac{1}{64}$ of its original value, or $(7.97 \times 10^{13})/(64) = 1.25 \times 10^{12}$ per second.

